NUMERICAL SOLUTION OF EQUATIONS IN THE 19TH CENTURY

17-18 JUNE 2019

Bergische Universität Wuppertal // Senatssaal (K.11.07)

At the croassroads of analysis, calculus, and algebra the numerical solving of algebraic equations gained more and more importance in the 19th century. This meeting focuses on the different approaches which characterise that period, notably those aiming at formulas and those aiming at algorithmic procedures. Previous practices that contributed to this status quo as well as the consideration of the immediate posterity complete the picture.

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Speakers

Sylvain Demanie (University of Lorraine)
Jean Dhombres (Centre Koyré/EHESS, Paris)
Massimo Galuzzi (University of Milan)
Thierry Joffredo (University of Lorraine)
Hourya Sinaceur (CNRS, Paris)
Dominique Tournès (University of La Réunion)
Cédric Vergnerie (University of Paris 7)
Yannick Vincent (École Polytechnique, Paris)

Organisation

Sara Confalonieri (University of Wuppertal)

BERGISCHE UNIVERSITÄT WUPPERTAL

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HISTOIRE DES MATHÉMATIQUES

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Numerical Solution of Equations in the 19th Century

Organised by Sara Confalonieri (Wuppertal)

– Monday 17 June 2019 – K.11.07 (Senatssaal) –

10:00 – Sara Confalonieri (Wuppertal): “Et je l’ai résolue par une méthode exacte et générale”*: Fourier’s Mistaken Method to Count the Number of Real Roots of a Polynomial (and to Separate them)*

11:15 – Break

11:30 – Jean Dhombres (Paris): *How to Present an Algebraically Minded Fourier?*

12:45 – Lunch

14:00 – Hourya Sinaceur (Paris): *The Rule of Signs: from Descartes to Sturm*

15:15 – Break

15:45 – Sylvain Demanie (Nancy): *Sturm’s Theorem in the French Journal* Les Nouvelles Annales de Mathématiques

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– Tuesday 18 June 2019 – K.11.07 (Senatssaal) –

9:00 – Massimo Galuzzi (Milano): *Lagrange and the Beauty of his Algorithms*

10:15 – Break

10:30 – Thierry Joffredo (Nancy): *Singular Points of Algebraic Curves: Rediscoveries of Newton's Parallelogram Method in the Second Half of the 19th Century*

11:45 – Dominique Tournès (La Réunion): *Effective Solution of Numerical Equations in 19th-Century Engineering*

13:00 – Lunch

14:15 – Yannick Vincent (Paris): *What’s About the Link Between Teaching and Research at the Ecole Polytechnique During the 19th Century ? An Example of the Numerical Equations*

BERGISCHE UNIVERSITÄT WUPPERTAL

Numerical Solution of Equations in the 19th Century

Summaries

Sara Confalonieri (Bergische Universität Wuppertal): "Et je l'ai résolue par une méthode exacte et générale"*: Fourier's Mistaken Method to Count the Number of Real Roots of a Polynomial (and to Separate them).*

It is little known that Fourier's method to count real roots of a polynomial provides at the end of the many cases and sub-cases the exact number of real roots. This is the same result achieved by Sturm; however, while the second result knew a fairly instant success, the first one was virtually ignored. Its partial reception in the contemporary mathematical community led to the assimilation with Budan's theorem, which, in turn, gives only an upper bound of the number of roots, just like the old rule of the signs of Descartes. The talk presents Fourier's method and seeks on different levels for a possible explanation of the inconsistency of its reception.

Sylvain Demanie (Archives Henri-Poincaré, Université de Lorraine): *Sturm's Theorem in the French Journal* Les Nouvelles Annales de Mathématiques.

By studying the circulation of Sturm's theorem in a mathematics journal devoted to teachers and students, we will highlight the propagation, use, and evolution of Sturm's theorem during the nineteenth century.

Jean Dhombres (Centre Koyré/EHESS, Paris): *How to Present an Algebraically Minded Fourier?*

Joseph Fourier (1768-1830) chose for a title to his opus magnum published in 1822: *The analytic theory of the propagation of heat*. Has here the adjective "analytic" the same meaning as in Lagrange's "Theory of analytic functions"? And what about *Mechanique analitique* (1788) of Lagrange? A posthumous book (1831) by Fourier, directly related to the present subject discussed in Wuppertal, does mention "*Analyse*" rather than analytic : *Analyse des équations déterminées*. And the same year a paper from Fourier appeared that concerned the "application of the principles of algebraic analysis" to "transcendantal equations", in fact numerical equations, but those that are not reduced to finding the roots of polynomials. By the way, some of these non polynomial equations were directly related to the famous problem of the temperature of the Earth, with in particular the greenhouse effect in the mind of everybody to-day. Everybody too knows that in keeping the expression "algebraic analysis" in his 1821 book, Cauchy was explicit in rejecting the generality attributed to algebra. Nevertheless Cauchy "proved" a theorem on functions that was obviously contradicted by the Fourier's example about the 2π-periodic step function, drawn as early as 1807. So my purpose, from a new study of Fourier's manuscript and as a sort of constructive game, will be to position Fourier as opposed to Cauchy in the task of using algebraic arguments to build Analysis. In particular to study proper modes, in particular Fourier series and integrals that were viewed in opposition to power series. This was not really understood before say functional analysis and abstract harmonic analysis in the 20th century, but Hilbert envisaged a posterity when he coined the expression generalized Fourier coefficients to describe a decomposition in a L^2 space.

Dhombres, Jean. De l'écriture des mathématiques en tant que technique de l'intellect, in Eric Guichard (dir.), *Ecritures : Sur les traces de Jack Goody*, Presses de l'ENSSIB, Lyon, 2012, pp. 157-198.

Dhombres, Jean. De Marin Mersenne à Joseph Fourier : La boîte à outils graphiques du physico-mathématicien, *Sciences et Techniques en Perspective*, IIe série, vol. 18, fasc. 2, 2016, p. 67-130.

Dhombres, Jean; Robert, Jean-Bernard, *Joseph Fourier, créateur de la physique mathématique*, Paris, Belin, 1998.

Fourier, Joseph, The manuscript entitled, Sur la propagation de la chaleur, kept at the Ecole nationale des Ponts et Chaussées (Ms 1851, 121 folios, recto and verso) has been transcribed and commented by Ivor Grattan-Guinness, in collaboration with J.R. Ravetz, *Joseph Fourier, 1768-1830. A survey of his life and work, based on a critical edition of his monograph on the propagation of heat, presented to the Institut de France in 1807*, MIT Press, Cambridge, Massachusetts, and London, England, 1972.

Fourier 1811, The manuscript sent by Fourier to get the « grand prix des sciences mathématiques », is kept in the Archives de l'Académie des sciences, and has been largely published in *Mémoires de l'Académie des sciences*, under the title, Théorie du mouvement de la chaleur dans les corps solides, vol. 4, 1824, p. 185-555, vol. 5, 1826, p. 231-320, and vol. 10, 1831, p. 119-146 (available on line).

Fourier, Joseph, Mémoire sur la température du globe terrestre et des espaces planétaires, *Annales de chimie et de physique*, 2e série, XXVII, p. 136-167 (available on line Bibnum, with a commentary by James Lequeux).

Fourier, Joseph, *The Analytical Theory of Heat*, translation of the 1822 book with notes by Alexander Freeman, Cambridge, at the University Press, 1878 (reprinted in 1955, New York), (available on line).

Kahane, Jean-Pierre; Lemarié-Rieusset,P.G., *Fourier series and wavelets*, Gordon and Breach, 1995, French version, *Séries de Fourier et ondelettes*, Cassini, 1998.

Massimo Galuzzi (Università degli Studi di Milano): *Lagrange and the Beauty of his Algorithms.*

Lagrange often dealt in his career with general equations, equations with literal coefficients or equations whose roots could be considered as indeterminates. The algorithms obtained for these equations, always very elegant, sometimes were judged to be ineffective by contemporaries, like his "Equation aux carrés des différences". In other case these, algorithms have had a long and varied life, like the famous "Série de Lagrange", which originally introduced to solve algebraic equations underwent important modifications and enrichments (by Laplace, the same Lagrange, Cauchy, etc.) and finally became an important tool for the theory of functions of a complex variable. The means of modern mathematics allow us to easily "revisit" these algorithms, not only to enjoy their beauty, but also to use them as useful tools for combinatorics.

Thierry Joffredo (Archives Henri-Poincaré, Université de Lorraine): *Singular Points of Algebraic Curves: Rediscoveries of Newton's Parallelogram Method in the Second Half of the 19th Century.*

After 1850, in England, Germany or France, some of the mathematiciens who are interested in algebraic curves and their singular points rediscover the Newton's parallelogram method, which seems then largely neglected, even forgotten, since the past century. "How completely it has dropped out of sight will appear from the uses which can be made of it, and which, it seems to me, must have been most obvious to any writer on curves, or on the theory of equations, who had really obtained possession of it.", said Augustus de Morgan, obviously surprised, in a lecture read in front of the members of the Cambridge Philosophical Society in 1855 and later published in the *Philosophical Transactions* under the title „On the Singular Points of Curves, and on Newton's Method of Coordinated Exponents". In this talk, we will shortly expose some of the works of these 19th century geometers on algebraic curves putting into action the Newton's parallelogram. We will therefore show that these new uses are mostly based on new readings of Gabriel Cramer's *Introduction à l'analyse des lignes courbes algébriques*, printed in Geneva in 1750, in which is made extensive use of this method to study infinite branches and singular points of curves, thus illustrating the continuities that exist between the 18th and 19th centuries in geometry.

Hourya Benis-Sinaceur (IHPST, Université Paris 1): *The Rule of Signs: from Descartes to Sturm.*

In this talk, I will explain how Sturm combined Descartes' rule of signs with a variant of Euclid's algorithm for polynomial in order to determine the exact number of real roots of a polynomial. Moreover, I describe the mathematical context of Sturm's discovery, showing the interweaving of the study of differential equations with algebraic questions.

Dominique Tournès (Université de La Réunion): *Effective Solution of Numerical Equations in 19th-Century Engineering.*

In 1876, the French engineer Léon-Louis Lalanne wrote about the solution of numerical equations that "finally it must be recognized that, while continuing to earn the admiration of geometers, the discoveries of Lagrange, Cauchy, Fourier, Sturm, Hermite, etc., did not always provide easily practicable means for the determination of the roots". To clarify the meaning of this statement, we propose to study some alternative methods, numerical or graphical, developed by 19th-century engineers and other practitioners to obtain effective solutions more adapted to the problems they had to deal with.

Y.annick Vincent (LinX - Ecole polytechnique): *What's About the Link Between Teaching and Research at the Ecole Polytechnique During the 19th Century? An Example of the Numerical Equations.*

During the nineteenth century, lots of well-known scientists taught at the Ecole Polytechnique. They sometimes taught their own research in their mathematical course. We will see how they made their students work on new topics around the issue of numerical equations. More generally, this study deals also with the freedom which was allowed to scientists in a military school at this period.